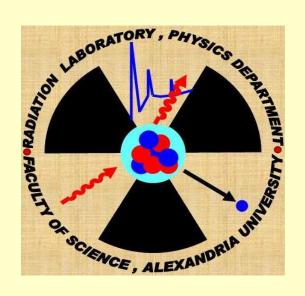
# سم الله الرحمن الرحيم

Phy 232
(Introduction to
Nuclear Physics)



Dr. Mohamed Salem

# ميثاق العمل خلال الحواس









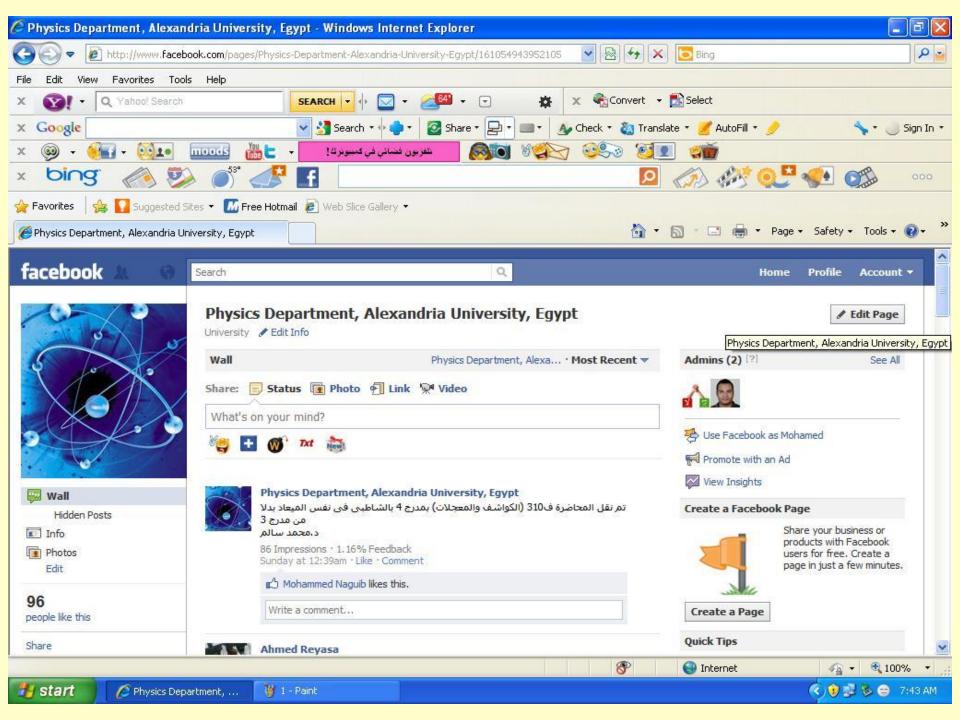


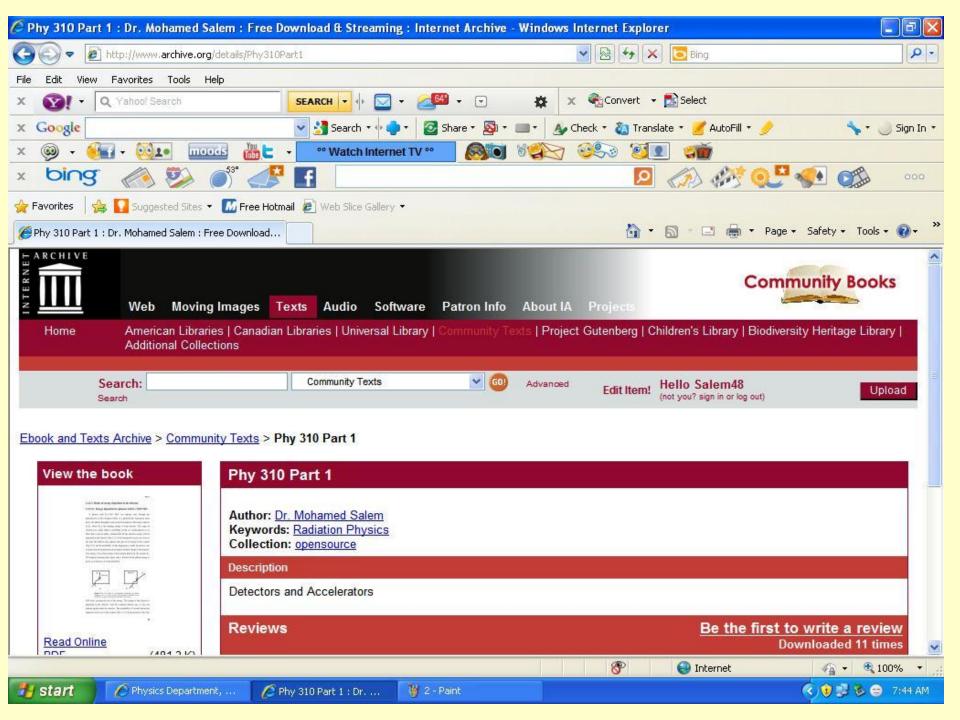


# المشامركات الايجابية مطلب مهمر

"أنني لن أقول لكم شيئًا بمفردي لكننا سوف نقول كل شيء معا"











# الثالمي الجمادي الأوائي في إجطار الإهمامات الوقي

معمل الفيزياء الأشعاعية (معمل الاستاذ الدكتور/ يونس صالح سليم)

قسم الفيزياء - كلية العلوم - جامعة الأسكندرية

الخريجي الكليات العلمية والعملية والعاملين في مجال الاشعاع

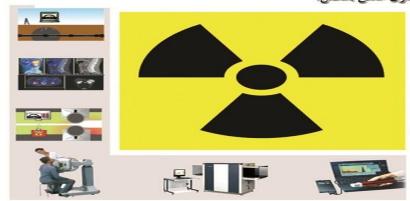
والطب والزراعة والصناعة والبحث العلمي ......

يعلن معمل الفيزياء الأشعاعية (معمل الاستاذ الدكتور/ يونس صالح سليم) - قسم الفيزياء - كلية العلوم - جامعة الأسكندرية عن عقد برنامج تدريمي في الوقاية من خطر الأشعاعات المؤينة لمستوى العلميين والفنيين (مساعد مصور اشعاعي /طبيب اشعة غير اخصائي/فيزيائي صحي/خيراء مؤهلين/مستخدمي النظائر المشعة) و المعتمدة من قبل وزارة الصحة - الادارة العامة للأشعة - المكتب التنفيذي للوقاية من الأشعة وذلك إبتداء من ٧ / / ١ / ٩ طبقا للجدول المعلن بالمعمل.

#### عميزات البرنامج التلوريي:

كل من يجتاز البرنامج بنجاج يحصل على شهادة معتمدة بموجبها له الحق في التقدم لوزارة الصحة لطلب ترخيص شخصي لمزاولة العمل في مجال الأشعاع في شتى الجالات و الهيئات الأتية:

- ا = هيئة الجمارك بالموانيء البحرية والمطارات.
- و عدات التصوير الطبي بالأشعة في المستشفيات والمراكز الطبية.
  - م = شركات التنقيب عن البترول والمعادن .
    - ع = الجامعات ومواكو البحوث.
  - ه = المصانع والمؤسسات القائمة على استخدام النظائر المشعة.
  - رسوم الأشتراك: ٥٥٥ جنيه للخريجين و٥٥٥ جنيه للطالاب
     (الخاضرات النظرية و التدريبات العملية).



للأستعلام: جامعة الأسكندرية الد. لحمد محمد احمد الخطيب (١٢٠ ٢٣٠١٠٠) د. محمد سالم محمد بدوى (٢٧٠٤٥١٥٠١٠)

رئيس مجلس قسم الفيزياء

منسق البرنامج والخبير الوقائي

١.د. محمد على عطية البرعي

فملاسعياء

۱.د. محمد اسماعیل عبده

عميد الكلية

ا.د.احد محمد احد الخطيب

# **The First Chapter**

#### **Atomic Structure.**

The atom is considered to be the basic building block of all matter. A simple theory of the atom tells us that it consists of two components: a nucleus surrounded by an electron cloud. The situation can be considered as being similar in some respects to planets orbiting the sun.

From an electrical point of view, the nucleus is said to be positively charged and the electrons negatively charged.

From a size point of view, the radius of an atom is about 10<sup>-10</sup> m while the radius of a nucleus is about 10<sup>-14</sup> m, i.e. about ten thousand times smaller. The situation could be viewed as something like a cricket ball, representing the nucleus, in the middle of a sporting arena with the electrons orbiting somewhere around where the spectators would sit. This perspective tells us that the atom should be composed mainly of empty space. However, the situation is far more complex than this simple picture portrays in that we must also take into account the physical forces which bind the atom together.

Chemical phenomena can be thought of as interactions between the electrons of individual atoms.

Radioactivity on the other hand can be thought of as changes which occur within the nuclei of atoms.

#### **Nuclear Structure**

A simple description of the nucleus tells us that it is composed of protons and neutrons. These two particle types are collectively called nucleons, i.e. particles which inhabit the nucleus.

From a mass point of view the mass of a proton is roughly equal to the mass of a neutron and each of these is about 2,000 times the mass of an electron. So most of the mass of an atom is concentrated in the small region at its core.

From an electrical point of view the proton is positively charged and the neutron has no charge. An atom all on its own (if that were possible to achieve!) is electrically neutral. The number of protons in the nucleus of such an atom must therefore equal the number of electrons orbiting that atom.

#### **Classification of Nuclei**

The term Atomic Number is defined in nuclear physics as the number of protons in a nucleus and is given the symbol Z. From your chemistry you will remember that this number also defines the position of an element in the Periodic Table of Elements.

The term Mass Number is defined as the number of nucleons in a nucleus, that is the number of protons plus the number of neutrons, and is given the symbol A.

It is possible for nuclei of a given element to have the same number of protons but differing numbers of neutrons, that is to have the same Atomic Number but different Mass Numbers. Such nuclei are referred to as Isotopes. All elements have isotopes and the number ranges from three for hydrogen to over 30 for elements such as cesium and barium.

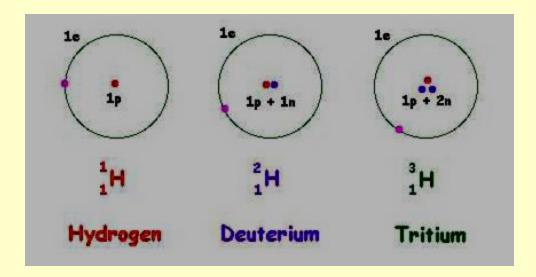
Chemistry has a relatively simple way of classifying the different elements by the use of symbols such as H for hydrogen, He for helium and so on. The classification scheme used to identify different isotopes is based on this approach with the use of a superscript before the chemical symbol to denote the Mass Number along with a subscript before the chemical symbol to denote the Atomic Number. In other words an isotope is identified as:



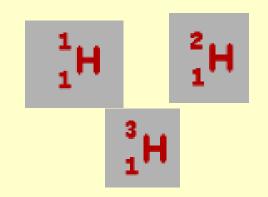
where **X** is the chemical symbol of the element.

Let us take the case of hydrogen as an example. It has three isotopes:

- 1. the most common one consisting of a single proton orbited by one electron,
- 2. a second isotope consisting of a nucleus containing a proton and a neutron orbited by one electron,
- 3. a third whose nucleus consists of one proton and two neutrons, again orbited by a single electron.



- •The first isotope commonly called hydrogen has a Mass Number of 1, an Atomic Number of 1.
- •The second isotope commonly called deuterium has a Mass Number of 2, an Atomic Number of 1.
- •The third isotope commonly called deuterium has a Mass Number of 3, an Atomic Number of 1.



Before we leave this classification scheme let us further consider the difference between chemistry and nuclear physics. You will remember that the water molecule is made up of two hydrogen atoms bonded with an oxygen atom. Theoretically if we were to combine atoms of hydrogen and oxygen in this manner many, many of billions of times we could make a glass of water. We could also make our glass of water using deuterium instead of hydrogen. This second glass of water would theoretically be very similar from a chemical perspective. However, from a physics perspective our second glass would be heavier than the first since each deuterium nucleus is about twice the mass of each hydrogen nucleus. Indeed water made in this fashion is called heavy water.

#### **Atomic Mass Unit**

The conventional unit of mass, the gram, is rather large for use in describing characteristics of nuclei. For this reason, a special unit called the Atomic Mass Unit (amu) is often used. This unit is sometimes defined as 1/12th of the mass of the stable most commonly occurring isotope of carbon, i.e. <sup>12</sup>C. In terms of grams, 1 amu is equal to  $1.66 \times 10^{-24}$  g, that is, just over one million, million million millionth of a gram.

The masses of the proton,  $m_p$  and neutron,  $m_n$  on this basis are:

 $m_p = 1.00783$  amu

and

 $m_n = 1.00866$  amu

while that of the electron is just 0.00055 amu.

# **Binding Energy**

We are now in a position to consider the subject of nuclear stability. From what we have covered so far, we have seen that the nucleus is a tiny region in the centre of an atom and that it is composed of neutrally and positively charged particles. So, in a large nucleus such as that of uranium (Z=92) we have a large number of positively charged protons concentrated into a tiny region in the centre of the atom. An obvious question which arises is that with all these positive charges in close proximity, how come the nucleus does not fly apart? How can a nucleus remain as an entity with such electrostatic repulsion between the components? Should the orbiting negatively-charged electrons not attract the protons away from the nucleus?

Let us take the case of the helium-4 nucleus as an example. This nucleus contains two protons and two neutrons so that in terms of amu we can figure out from what we covered earlier that the

mass of 2 protons = 2.01566 amu,

and the

mass of 2 neutrons = 2.01732 amu.

Therefore we would expect the total mass of the nucleus to be 4.03298 amu.

The experimentally determined mass of a helium-4 nucleus is a bit less - just 4.00260 amu. In other words there is a difference of 0.03038 amu between what we might expect as the mass of this nucleus and what we actually measure. You might think of this difference as very small at just 0.75%.

But remember that since the mass of one electron is 0.00055 amu the difference is actually equivalent to the mass of about 55 electrons. Therefore it is significant enough to wonder about.

It is possible to consider that this missing mass is converted to energy which is used to hold the nucleus together, that is it is converted to a form of energy called the Binding Energy. You could say like all relationships, energy must be expended in maintaining them!

Like the gram in terms of the mass of nuclei, the common unit of energy, the joule is rather cumbersome when we consider the energy needed to bind a nucleus together. The unit used to express energies on the atomic scale is the electron volt, symbol: eV.

Albert Einstein introduced us to the equivalence of mass, m, and energy, E, at the atomic level using the following equation:

$$E = m c^2$$
,

where c is the velocity of light.

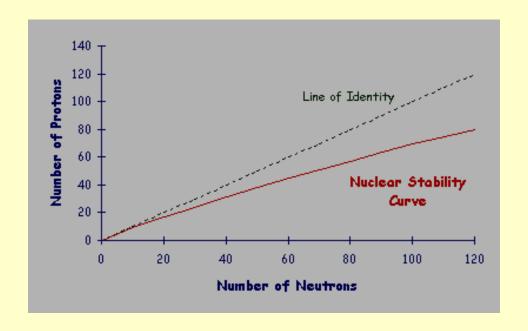
It is possible to show that 1 amu is equivalent to 931.48 MeV. Therefore, the mass difference we discussed earlier between the expected and measured mass of the helium-4 nucleus of 0.03038 amu is equivalent to about 28 MeV. This represents about 7 MeV for each of the four nucleons contained in the nucleus.

# **Nuclear Stability**

In most stable isotopes the binding energy per nucleon lies between 7 and 9 MeV. Since this binding energy is dependent on the number of nucleons in a nucleus, that is the Mass Number, A, and since the electrostatic repulsion between the protons is dependent on the square of the charges, that is  $\mathbb{Z}^2$ , it is possible to infer that  $\mathbb{Z}^2$  must be dependent on A for a nucleus to remain stable.

In other words to combat the effect of the increase in electrostatic repulsion when the number of protons increases the number of neutrons must increase more rapidly to contribute sufficient energy to bind the nucleus together.

As we noted earlier there are a number of isotopes for each element of the Periodic Table. It has been found that the most stable isotope for each element has a specific number of neutrons in its nucleus. Plotting a graph of the number of protons against the number of neutrons for these stable isotopes generates what is called the Nuclear Stability Curve:



Note that the number of protons equals the number of neutrons for small nuclei. But notice also that the number of neutrons increases more rapidly than the number of protons as the size of the nucleus gets bigger so as to maintain the stability of the nucleus. In other words, more neutrons need to be there to contribute to the binding energy used to counteract the electrostatic repulsion between the protons

# Radioactivity

There are about 2,450 known isotopes of the one hundred odd elements in the Periodic Table. You can imagine the size of a table of isotopes relative to that of the Periodic Table! The unstable isotopes lie above or below the Nuclear Stability Curve. These unstable isotopes attempt to reach the stability curve by splitting into fragments, in a process called Fission, or by emitting particles and/or energy in the form of radiation. This latter process is called Radioactivity.

It is useful to dwell for a few moments on the term radioactivity. For example what has nuclear stability to do with radio? From a historical perspective remember that when these radiations were discovered about 100 years ago we did not know exactly what we were dealing with. When people like Henri Becquerel and Marie Curie were working initially on these strange emanations from certain natural materials it was thought that the radiations were somehow related to another phenomenon which also was not well understood at the time - that of radio communication. It seems reasonable on this basis to appreciate that some people considered that the two phenomena were somehow related and hence that the materials which emitted radiation were termed *radio-active*.

We know today that the two phenomena are not directly related but we nevertheless hold onto the term radioactivity for historical purposes. But it should be quite clear to you having reached this stage of this chapter that the term radioactive refers to the emission of particles and/or energy from unstable isotopes. Unstable isotopes for instance those that have too many protons to remain a stable entity are called radioactive isotopes - and called radioisotopes for short. The term radionuclide is also sometimes used.

Finally about 300 of the 2,450-odd isotopes mentioned above are found in nature. The rest are man-made (or person-made!), that is they are produced artificially. These 2,150 or so artificial isotopes have been made during the last 100 years or so with most having been made since the second world war.

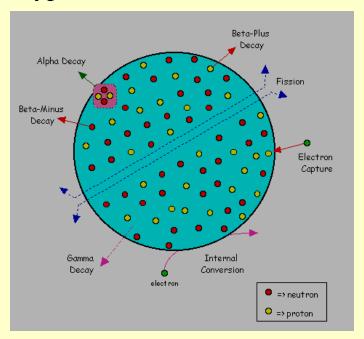
# **The Second Chapter**

#### Introduction

We saw in the last chapter that radioactivity is a process used by unstable nuclei to achieve a more stable situation. It is said that such nuclei *decay in* an attempt to achieve stability. So, an alternative title for this chapter is Nuclear Decay Processes.

# **Methods of Radioactive Decay**

Rather than considering what happens to individual nuclei it is perhaps easier to consider a hypothetical nucleus that can undergo many of the major forms of radioactive decay. This hypothetical nucleus is shown below:



- 1. Firstly we can see two protons and two neutrons being emitted together in a process called alpha decay.
- 2. Secondly, we can see that a proton can release a particle in a process called beta-plus decay, and that a neutron can emit a particle in a process called beta-minus decay.
- 3. We can also see an electron being captured by a proton.
- 4. Thirdly we can see some energy being emitted which results from a process called gamma-decay as well as an electron being attracted into the nucleus and being ejected again.
- 5. Finally there is the rather catastrophic process where the nucleus cracks in half called spontaneous fission. We will now describe each of these decay processes in turn.

# **Spontaneous Fission**

This is a very destructive process which occurs in some heavy nuclei which split into 2 or 3 fragments plus some neutrons. These fragments form new nuclei which are usually radioactive. Nuclear reactors exploit this phenomenon for the production of radioisotopes. Its also used for nuclear power generation and in nuclear weaponry. The process is not of great interest to us here and we will say no more about it for the time being.

# **Alpha Decay**

In this decay process two protons and two neutrons leave the nucleus together in an assembly known as an alpha particle. Note that an alpha particle is really a helium-4 nucleus.

So why not call it a helium nucleus? Why give it another name? The answer to this question lies in the history of the discovery of radioactivity. At the time when these radiations were discovered we didn't know what they really were. We found out that one type of these radiations had a double positive charge and it was not until sometime later that we learnt that they were in fact nuclei of helium-4.

In the initial period of their discovery this form of radiation was given the name alpha rays (and the other two were called beta and gamma rays), these terms being the first three letters of the Greek alphabet. We still call this form of radiation by the name alpha particle for historical purposes. Calling it by this name also contributes to the specific jargon of the field and leads outsiders to think that the subject is quite specialized!

But notice that the radiation really consists of a helium-4 nucleus emitted from an unstable larger nucleus. There is nothing strange about helium since it is quite an abundant element on our planet. So why is this radiation dangerous to humans? The answer to this question lies with the energy with which they are emitted and the fact that they are quite massive and have a double positive charge. So when they interact with living matter they can cause substantial destruction to molecules which they encounter in their attempt to slow down and to attract two electrons to become a neutral helium atom.

An example of this form of decay occurs in the uranium-238 nucleus. The equation which represents what occurs is:

$$_{92}^{238}U \rightarrow_{90}^{234}Th + _{2}^{4}He$$

Here the uranium-238 nucleus emits a helium-4 nucleus (the alpha particle) and the parent nucleus becomes thorium-234. Note that the Mass Number of the parent nucleus has been reduced by 4 and the Atomic Number is reduced by 2 which is a characteristic of alpha decay for any nucleus in which it occurs.

### **Beta Decay**

There are three common forms of **beta decay**:

## (a) Electron Emission

Certain nuclei which have an excess of neutrons may attempt to reach stability by converting a neutron into a proton with the emission of an electron. The electron is called a beta-minus particle - the minus indicating that the particle is negatively charged.

We can represent what occurs as follows:

electric charge is conserved.

where a neutron converts into a proton and an electron. Notice that the total electrical charge is the same on both sides of this equation. We say that the

 $\mathbf{n}^{\circ} \rightarrow \mathbf{p}^{+} + \mathbf{e}^{-}$ 

We can consider that the electron cannot exist inside the nucleus and therefore is ejected.

Once again there is nothing strange or mysterious about an electron. What is important though from a radiation safety point of view is the energy with which it is emitted and the chemical damage it can cause when it interacts with living matter.

An example of this type of decay occurs in the iodine-131 nucleus which decays into xenon-131 with the emission of an electron, that is

The electron is what is called a beta-minus particle. Note that the Mass Number in the above equation remains the same and that the Atomic Number increases by 1 which is characteristic of this type of decay.

### (b) Positron Emission

When the number of protons in a nucleus is too large for the nucleus to be stable it may attempt to reach stability by converting a proton into a neutron with the emission of a positively-charged electron.

That is not a typographical error! An electron with a positive charge also called a positron is emitted. The positron is the beta-plus particle.

The history here is quite interesting. A brilliant Italian physicist, Enrico Fermi developed a theory of beta decay and his theory predicted that positively-charged as well as negatively-charged electrons could be emitted by unstable nuclei. These particles could be called pieces of anti-matter and they were subsequently discovered by experiment. They do not exist for very long as they quickly combine with a normal electron and the subsequent reaction called annihilation gives rise to the emission of two gamma rays.

The reaction in our unstable nucleus which contains one too many protons can be represented as follows:  $p^+ \rightarrow n^0 + e^+$ 

Notice, once again, that electric charge is conserved on each side of this equation.

An example of this type of decay occurs in sodium-22 which decays into neon-22 with the emission of a positron:

Note that the Mass Number remains the same and that the Atomic Number decreases by 1.

# c) Electron Capture

In this third form of beta decay an inner orbiting electron is attracted into an unstable nucleus where it combines with a proton to form a neutron. The reaction can be represented as:  $e^- + p^+ \rightarrow n^0$ 

This process is also known as K-capture since the electron is often attracted from the K-shell of the atom.

How do we know that a process like this occurs? The signature of this type of decay can be obtained from effects in the electron cloud surrounding the nucleus when the vacant site left in the K-shell is filled by an electron from an outer shell. The filling of the vacancy is associated with the emission of an X-ray from the electron cloud and it is this X-ray which provides a signature for this type of beta decay.

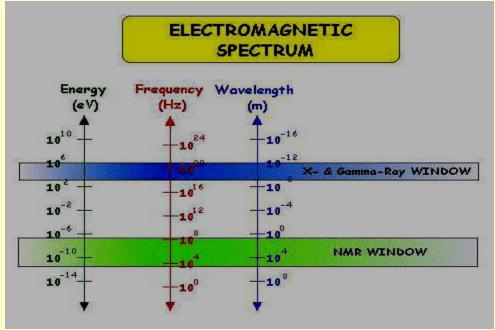
Note that the Mass Number once again is unchanged in this form of decay and that the Atomic Number is decreased by 1.

# Gamma Decay

Gamma decay involves the emission of energy from an unstable nucleus in the form of electromagnetic radiation.

The radiation can be characterized in terms of its frequency, its wavelength and its energy. Thinking about it in terms of the energy of the radiation we have very low energy electromagnetic radiation called radio waves, infra-red radiation at a slightly higher energy, visible light at a higher energy still, then ultra-violet radiation and the higher energy forms of this radiation are called X-rays and gamma-rays. You should also remember that these radiations form what is called the

Electromagnetic Spectrum.



Before proceeding it is useful to pause for a moment to consider the difference between X-rays and gamma-rays. These two forms of radiation are high energy electromagnetic rays and are therefore virtually the same. The difference between them is not what they consist of but where they come from.

In general we can say that if the radiation emerges from a nucleus it is called a gamma-ray and if it emerges from outside the nucleus from the electron cloud for example, it is called an X-ray.

There are two common forms of gamma decay:

### (a) Isomeric Transition

A nucleus in an excited state may reach its ground or unexcited state by the emission of a gamma ray.

An example of this type of decay is that of technetium-99m - which by the way is the most common radioisotope used for diagnostic purposes today in medicine. The reaction can be expressed as:

Here a nucleus of technetium-99 is in an excited state, that is it has excess energy. The excited state in this case is called a metastable state and the nucleus is therefore called technetium-99m (m for metastable). The excited nucleus looses its excess energy by emitting a gamma-ray to become technetium-99.

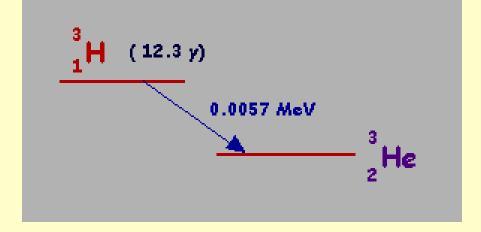
# (b) Internal Conversion

Here the excess energy of an excited nucleus is given to an atomic electron, e.g. a K-shell electron.

## **Decay Schemes**

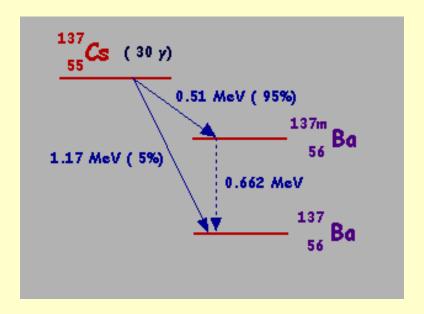
Decay schemes are widely used to give a visual representation of radioactive decay. A scheme for a relatively straight-forward decay is shown

below:



This scheme is for hydrogen-3 which decays to helium-3 with a half-life of 12.3 years through the emission of a beta-minus particle with an energy of 0.0057 MeV.

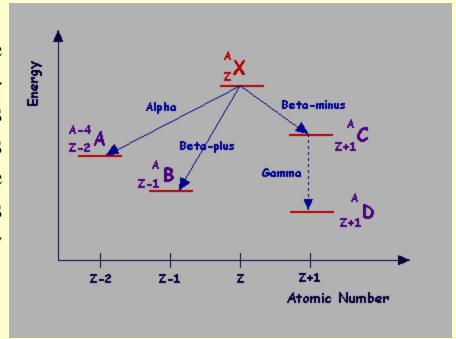
A scheme for a more complicated decay is that of caesium-137:



This isotope can decay through two beta-minus processes. In one which occurs in 5% of disintegrations a beta-minus particle is emitted with an energy of 1.17 MeV to produce barium-137. In the second which occurs more frequently (in the remaining 95% of disintegrations) a beta-minus particle of energy 0.51 MeV is emitted to produce barium-137m - in other words a barium-137 nucleus in a metastable state. The barium-137m then decays via isomeric transition with the emission of a gamma-ray of energy 0.662 MeV.

The general method used for decay schemes is illustrated in the diagram below:

The isotope from which the scheme originates is displayed at the top - X in the case above. This isotope is referred to as the parent. The parent looses energy when it decays and hence the products of the decay referred to as daughters are plotted at a lower energy level.



The diagram illustrates the situation for common forms of radioactive decay. Alpha-decay is illustrated on the left where the mass number is reduced by 4 and the atomic number is reduced by 2 to produce daughter **A**. To its right the scheme for beta-plus decay is shown to produce daughter **B**. The situation for beta-minus decay followed by gamma-decay is shown on the right side of the diagram where daughters **C** and **D** respectively are produced.

# **The Third Chapter**

The Radioactive Decay Law

we will try here to relate the subject of radioactive decay to a more common situation which we will use as an analogy and hopefully we will be able to overcome the abstract feature of the subject matter. The analogy we will use here is that of making popcorn.

So think about putting some oil in a pot, adding the corn, heating the pot on the cooker and watching what happens. You might also like to try this out while considering the situation!

For our radioactive decay situation we first of all consider that we have a sample containing a large number of radioactive nuclei all of the same kind. This is our unopposed corn in the pot for example.

Secondly we assume that all of the radioactive nuclei decay by the same process be it alpha, beta or gamma-decay. In other words our unopposed corn goes pop at some stage during the heating process.

Thirdly take a few moments to ponder on the fact that we can only really consider what is going on from a statistical perspective. If you look at an individual piece of corn, can you figure out when it is going to pop? Not really. You can however figure out that a large number of them will have popped after a period of time. But its rather more difficult to figure out the situation for an individual piece of corn. So instead of dealing with individual entities we consider what happens on a larger scale and this is where statistics comes in. We can say that the radioactive decay is a statistical one-shot process, that is when a nucleus has decayed it cannot repeat the process again. In other words when a piece of corn has popped it cannot repeat the process. Simple!

In addition as long as a radioactive nucleus has not decayed the probability for it doing so in the next moment remains the same. In other words if a piece of corn has not popped at a certain time the chance of it popping in the next second is the same as in the previous second.

Let us not push this popcorn analogy too far though in that we know that we can control the rate of popping by the heat we apply to the pot for example. However as far as our radioactive nuclei are concerned there is nothing we can do to control what is going on. The rate at which nuclei go pop (or decay, in other words) cannot be influenced by heating up the sample. Nor by cooling it for that matter or by putting it under greater pressures, by changing the gravitational environment by taking it out into space for instance, or by changing any other aspect of its physical environment. The only thing that determines whether an individual nucleus will decay seems to be the nucleus itself. But on the average we can say that it will decay at some stage.

## The Radioactive Decay Law

Let us now use some symbols to reduce the amount of writing we have to do to describe what is going on and to avail ourselves of some mathematical techniques to simplify the situation even further than we have been able to do so far.

Let us say that in the sample of radioactive material there are N nuclei which have not decayed at a certain time, t. So what happens in the next brief period of time? Some nuclei will decay for sure. But how many?

On the basis of our reasoning above we can say that the number which will decay will depend on overall number of nuclei, N, and also on the length of the brief period of time. In other words the more nuclei there are the more will decay and the longer the time period the more nuclei will decay. Let us denote the number which will have decayed as dN and the small time interval as dt.

So we have reasoned that the number of radioactive nuclei which will decay during the time interval from t to t+dt must be proportional to N and to dt. In symbols therefore:

–dN ∝ N.dt

the minus sign indicating that N is decreasing.

Turning the proportionality in this equation into an equality we can write:

$$-dN = \lambda N.dt$$

where the constant of proportionality,  $\lambda$ , is called the Decay Constant. Dividing across by N we can rewrite this equation as:

$$-\frac{dN}{N} = \lambda . dt$$

So this equation describes the situation for any brief time interval, dt. To find out what happens for all periods of time we simply add up what happens in each brief time interval. In other words we integrate the above equation. Expressing this more formally we can say that for the period of time from t=0 to any later time t, the number of radioactive nuclei will decrease from  $N_0$  to  $N_t$ , so that:

$$-\int_{N_0}^{N_t} \frac{dN}{N} = \lambda_0^t dt$$

$$\therefore \ln\left(\frac{N_t}{N_0}\right) = -\lambda t$$

$$\therefore \frac{\mathbf{N_t}}{\mathbf{N_0}} = \exp (-\lambda t)$$

$$\therefore \mathbf{N}_{t} = \mathbf{N}_{0} \exp (-\lambda t)$$

This final expression is known as the Radioactive Decay Law. It tells us that the number of radioactive nuclei will decrease in an exponential fashion with time with the rate of decrease being controlled by the Decay Constant.

Before looking at this expression in further detail let us review the mathematics which we used above. First of all we used integral calculus to figure out what was happening over a period of time by integrating what we knew would occur in a brief interval of time. Secondly we used a calculus relationship that the

$$\int \frac{dx}{x} = \ln x$$

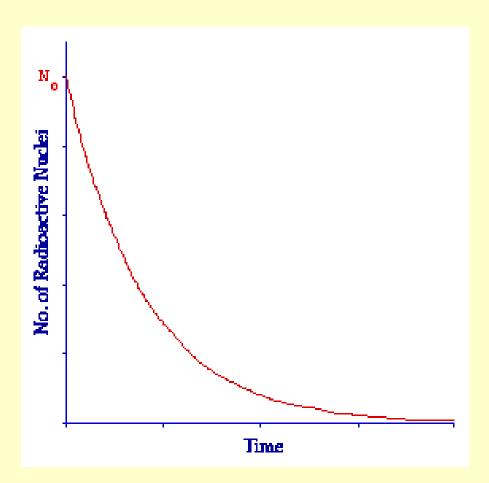
where ln x represents the natural logarithm of x. And thirdly we used the definition of logarithms that when

$$\ln x = y$$

then,

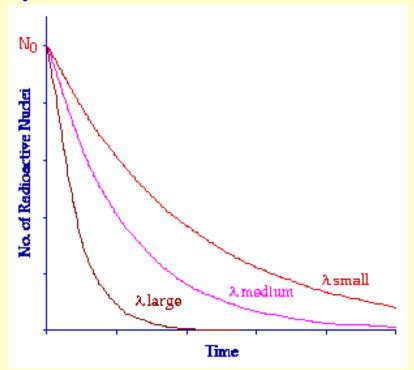
$$x = exp y$$

Now, to return to the Radioactive Decay Law. The Law tells us that the number of radioactive nuclei will decrease with time in an exponential fashion with the rate of decrease being controlled by the Decay Constant. The Law is shown in graphical form in the figure below:



The graph plots the number of radioactive nuclei at any time,  $N_t$ , against time, t. We can see that the number of radioactive nuclei decreases from  $N_0$  that is the number at t=0 in a rapid fashion initially and then more slowly in the classic exponential manner.

The influence of the Decay Constant can be seen in the following figure:



All three curves here are exponential in nature, only the Decay Constant is different. Notice that when the Decay Constant has a low value the curve decreases relatively slowly and when the Decay Constant is large the curve decreases very quickly.

The Decay Constant is characteristic of individual radionuclide's. Some like uranium-238 have a small value and the material therefore decays quite slowly over a long period of time. Other nuclei such as technetium-99m have a relatively large Decay Constant and they decay far more quickly.

It is also possible to consider the Radioactive Decay Law from another perspective by plotting the logarithm of  $N_t$  against time. In other words from our analysis above by plotting the expression:

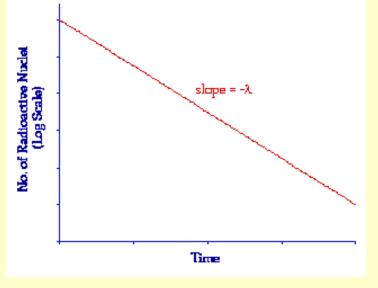
 $\ln\left(\frac{N_t}{N_0}\right) = -\lambda t$ 

in the form

$$ln N_t = -\lambda t + ln N_0$$

Notice that this expression is simply an equation of the form y = mx + c where  $m = -\lambda$  and  $c = \ln N_0$ . As a result it is the equation of a straight line of slope  $-\lambda$  as shown in the following figure. Such a plot is sometimes useful when we wish to consider a situation without the complication of the direct exponential

behavior.

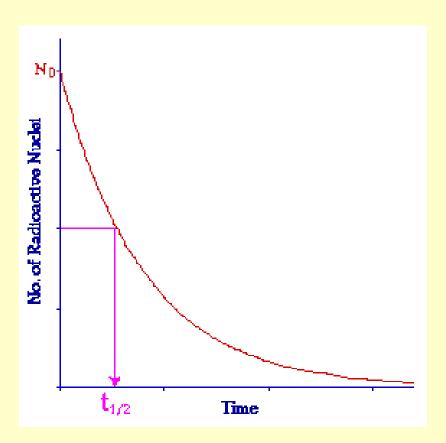


### Half-Life

The Half Life and it expresses the length of time it takes for the radioactivity of a radioisotope to decrease by a factor of two. From a graphical point of view we can say that when:

 $N_t = \frac{N_0}{2}$ 

the time taken is the Half Life:



Radioisotope	Half Life (approx.)	
81mKr	13 seconds	
<sup>99m</sup> Te	6 hours	
131 <sub>I</sub>	8 days	
<sup>51</sup> Cr	1 month	
137 <sub>Cs</sub>	30 years	
<sup>241</sup> Am	462 years	
<sup>226</sup> Ra	1620 years	
238 <sub>U</sub>	4.51 x 10 <sup>9</sup> years	

## Relationship between the Decay Constant and the Half Life

When the Decay Constant is small the Half Life should be long and correspondingly when the Decay Constant is large the Half Life should be short. But what exactly is the nature of this relationship?

We can easily answer this question by using the definition of Half Life and applying it to the Radioactive Decay Law.

Once again the law tells us that at any time, t:

$$N_t = N_0 \exp(-\lambda t)$$

and the definition of Half Life tells us that:

When

$$N_t = \frac{N_0}{2}$$

We can therefore re-write the Radioactive Decay Law by substituting for  $N_t$  and t as follows:

Therefore

$$\frac{N_0}{2} = N_0 \exp(-\lambda t_{\frac{1}{2}})$$

$$\therefore 2^{-1} = \exp(-\lambda t_{\frac{1}{2}})$$

$$\therefore \ln 2^{-1} = -\lambda t_{\frac{1}{2}}$$

$$\therefore \ln 2 = \lambda t_{\frac{1}{2}}$$

$$\therefore 0.693 = \lambda t_{\frac{1}{2}}$$

$$\lambda = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

These last two equations express the relationship between the Decay Constant and the Half Life.

## **Units of Radioactivity**

The SI or metric unit of radioactivity is named after Henri Becquerel, in honor of his discovery of radioactivity, and is called the becquerel with the symbol Bq. The becquerel is defined as the quantity of radioactive substance that gives rise to a decay rate of 1 decay per second.

In medical diagnostic work 1 Bq is a rather small amount of radioactivity. Indeed it is easy to remember its definition if you think of it as a buggerall amount of radioactivity. For this reason the kilobecquerel (kBq) and megabecquerel (MBq) are more frequently used.

The traditional unit of radioactivity is named after Marie Curie and is called the curie, with the symbol Ci. The curie is defined as the amount of radioactive substance which gives rise to a decay rate of 3.7 x  $10^{10}$  decays per second. In other words 37 thousand, million decays per second which as you might appreciate is a substantial amount of radioactivity. For medical diagnostic work the millicurie (mCi) and the microcurie ( $\mu$ Ci) are therefore more frequently used.

## **Question 1**

- (a) The half-life of 99<sup>m</sup>Tc is 6 hours. After how much time will 1/16th of the radioisotope remain?
- (b) Verify your answer by another means.

#### **Answer:**

(a) Starting with the relationship we established earlier between the Decay Constant and the Half Life we can calculate the Decay Constant as follows:

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{6} = 0.1155 \text{ hr}^{-1}$$

Now applying the Radioactive Decay Law,  $N_t = N_0 \exp(-\lambda t)$  we can re-write it in the form:

The question tells us that  $N_0$  has reduced to 1/16th of its value, that is:

$$\frac{N_t}{N_0} = \exp (-\lambda t)$$

Therefore

$$\frac{1}{16} = \exp (-0.1155t)$$

which we need to solve for t. One way of doing this is as follows:

$$16^{-1} = \exp (-0.1155t)$$

$$t = \frac{\ln 16}{0.1155} = 24 \text{ hours}$$

So it will take 24 hours until 1/16th of the radioactivity remains.

(b) A way in which this answer can be verified is by using the definition of Half Life. We are told that the Half Life of 99<sup>m</sup>Tc is 6 hours. Therefore after six hours half of the radioactivity remains.

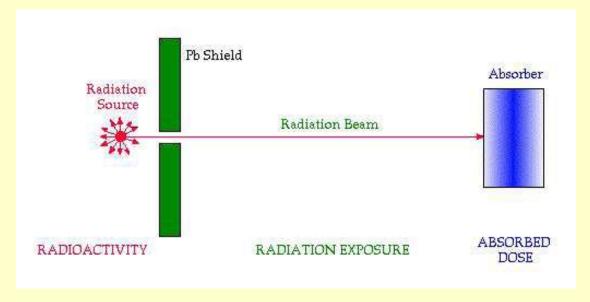
Therefore after 12 hours a quarter remains; after 18 hours an eighth remains and after 24 hours one sixteenth remains. And we arrive at the same answer as in part (a). So we must be right!

# **The Fourth Chapter**

**Units of Radiation Measurement** 

## **A Typical Radiation Situation**

A typical radiation set-up is shown in the figure below. Firstly there is a source of radiation, secondly a radiation beam and thirdly some material which absorbs the radiation. So the quantities which can be measured are associated with the source, the radiation beam and the absorber.



**The Radiation Source** 

When the radiation source is a radioactive one the quantity that is typically measured is the radioactivity of the source. We saw in the previous chapter that the units used to express radioactivity are the Becquerel (SI unit) and the curie (traditional unit).

#### The Radiation Beam

A straight-forward way of measuring such ionization is to determine the amount of electric charge which is produced. You will remember from your high school physics that the SI unit of electric charge is the coulomb.

The SI unit of radiation exposure is the coulomb per kilogram - and is given the symbol C kg<sup>-1</sup>. It is defined as the quantity of X- or gamma-rays such that the associated electrons emitted per kilogram of air at standard temperature and pressure (STP) produce ions carrying 1 coulomb of electric charge.

The traditional unit of radiation exposure is the roentgen, named in honor of Wilhelm Roentgen (who discovered X-rays) and is given the symbol R. The roentgen is defined as the quantity of X- or gamma-rays such that the associated electrons emitted per kilogram of air at STP produce ions carrying 2.58 x 10<sup>-4</sup> coulombs of electric charge.

So 1 R is a small exposure relative to 1 C kg<sup>-1</sup> - in fact it is 3,876 times smaller. Note that this unit is confined to radiation beams consisting of X-rays or gamma-rays.

Often it is not simply the exposure that is of interest but the exposure rate, that is the exposure per unit time. The units which tend to be used in this case are the  $C kg^{-1} s^{-1}$  and the  $R hr^{-1}$ .

#### The Absorber

Energy is deposited in the absorber when radiation interacts with it. It is usually quite a small amount of energy but energy nonetheless. The quantity that is measured is called the Absorbed Dose and it is of relevance to all types of radiation be they X- or gamma-rays, alpha- or beta-particles.

The SI unit of absorbed dose is called the gray, named after a famous radiobiologist, LH Gray, and is given the symbol Gy. The gray is defined as the absorption of 1 joule of radiation energy per kilogram of material. So when 1 joule of radiation energy is absorbed by a kilogram of the absorber material we say that the absorbed dose is 1 Gy.

The traditional unit of absorbed dose is called the rad, which supposedly stands for Radiation Absorbed Dose. It is defined as the absorption of 10<sup>-2</sup> joules of radiation energy per kilogram of material. As you can figure out 1 Gy is equal to 100 rad.

There are other quantities derived from the gray and the rad which express the biological effects of such absorbed radiation energy when the absorber is living matter - human tissue for example. These quantities include the Equivalent Dose and the Effective Dose.

## **Specific Gamma Ray Constant**

It is quite a useful quantity from a practical viewpoint when we are dealing with a radioactive source which emits gamma-rays. Supposing you are using a gamma-emitting radioactive source (for example <sup>99m</sup>Tc or <sup>137</sup>Cs) and you will be standing at a certain distance from this source while you are working. You most likely will be interested in the exposure rate produced by the source from a radiation safety point of view. This is where the Specific Gamma-Ray Constant comes in.

It is defined as the exposure rate per unit activity at a certain distance from a source. The SI unit is therefore the

$$C kg^{-1} s^{-1} Bq^{-1} at 1 m$$
,

and the traditional unit is the

R hr<sup>-1</sup> mCi<sup>-1</sup> at 1 cm.

# The Fifth Chapter

**Interaction of Radiation with Matter** 

## The Major Types of Radiation

Radiation	Mass	Electric Charge	Velocity
Alpha Particles	relatively heavy	double positive	relatively slow
Beta Particles	about 8,000 times lighter	negative	less than the velocity of light
Gamma Rays	None	None	3 x 10 <sup>8</sup> m/s in free space

We will now consider the passage of each type of radiation through matter with most attention given to gamma-rays because they are the most common type used in nuclear medicine. One of the main effects that you will notice irrespective of the type of radiation is that ions are produced when radiation interacts with matter. It is for this reason that it is called ionizing radiation.

## **Alpha Particles**

We can see from the table above that alpha-particles have a double positive charge and we can therefore easily appreciate that they will exert considerable electrostatic attraction on the outer orbital electrons of atoms near which they pass. The result is that some electrons will be attracted away from their parent atoms and that ions will be produced. In other words ionizations occur.

We can also appreciate from the table that alpha-particles are quite massive relative to the other types of radiation and also to the electrons of atoms of the material through which they are passing. As a result they travel in straight lines through matter except for rare direct collisions with nuclei of atoms along their path.

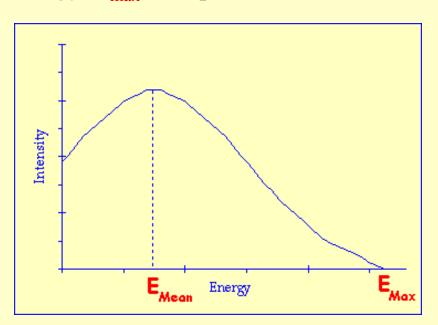
A third feature of relevance here is the energy with which they are emitted. This energy in the case of alpha-particles is always distinct. For example <sup>221</sup>Ra emits an alpha-particle with an energy of 6.71 MeV. Every alpha-particle emitted from this radionuclide has this energy. Another example is <sup>230</sup>U which emits three alpha-particles with energies of 5.66, 5.82, 5.89 MeV.

Finally it is useful to note that alpha-particles are very damaging biologically and this is one reason why they are not used for in-vivo diagnostic studies.

### **Beta Particles**

Because of their negative charge they are attracted by nuclei and repelled by electron clouds as they pass through matter. The result once again without going into great detail is ionization.

A final and important point to note is that the energy of beta-particles is never found to be distinct in contrast to the alpha-particles above. The energies of the beta-particles from a radioactive source forms a spectrum up to a maximum energy. Notice from the figure that a range of energies is present and features such as the mean energy,  $E_{mean}$ , or the maximum energy,  $E_{max}$ , are quoted.

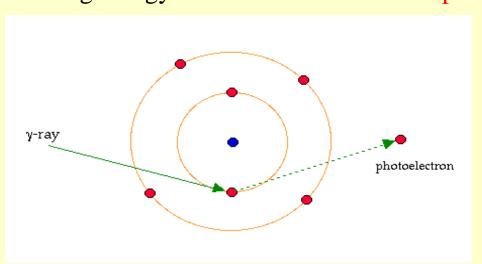


## Gamma Rays

Since we have been talking about energies above let us first note that the energies of gamma-rays emitted from a radioactive source are always distinct. For example <sup>99m</sup>Tc emits gamma-rays which all have an energy of 140 keV and <sup>51</sup>Cr emits gamma-rays which have an energy of 320 keV. Gamma-rays have many modes of interaction with matter. Those which are very important to nuclear medicine imaging, are the Photoelectric Effect and the Compton Effect. We will consider each of these in turn below.

### **Photoelectric Effect**

When a gamma-ray collides with an orbital electron of an atom of the material through which it is passing it can transfer all its energy to the electron and cease to exist. On the basis of the Principle of Conservation of Energy we can deduce that the electron will leave the atom with a kinetic energy equal to the energy of the gamma-ray less that of the orbital binding energy. This electron is called a photoelectron.



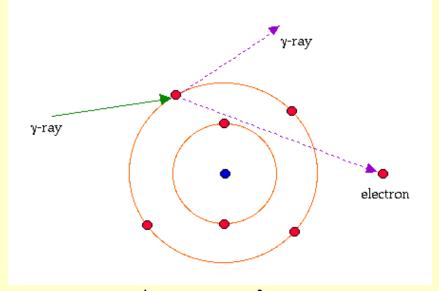
Note that an ion results when the photoelectron leaves the atom. Also note that the gamma-ray energy is totally absorbed in the process.

Two subsequent points should also be noted. Firstly the photoelectron can cause ionizations along its track in a similar manner to a beta-particle. Secondly X-ray emission can occur when the vacancy left by the photoelectron is filled by an electron from an outer shell of the atom.

## **Compton Effect**

This type of effect is somewhat akin to a cue ball hitting a colored ball on a pool table. Here a gamma-ray transfers only part of its energy to a valance electron which is essentially free. Notice that the electron leaves the atom and may act like a beta-particle and that the gamma-ray deflects off in a different direction to that with which it approached the atom. This deflected or scattered gamma-ray can undergo further Compton Effects within the material. Note that this effect is sometimes called

Compton Scattering.



Attenuation

The two effects we have just described give rise to both absorption and scattering of the radiation beam. The overall effect is referred to as attenuation of gamma-rays.